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A Higher Dimensional Inflationary Universe in General Relativity

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Abstract A five dimensional Kaluza-Klein inflationary universe is investigated in the presence of massless scalar field with a flat potential. To get an inflationary universe a flat region in which potential V is constant is considered. Some physical and kinematical properties of the universe are also discussed.

Keywords Higher dimensional · Inflationary universe · General relativity

1 Introduction

The standard explanation for the flatness of the universe is that it has undergone at an early stage a period of exponential expansion, known as inflation. The proposed inflationary universe scenario explains several of the mysteries of modern cosmology like the homogeneity, the isotropy and the flatness of the observed universe. Several authors [1–5] have investigated different aspects of the inflationary universe in general relativity.

Classical scalar fields are essential in the study of present day cosmological models. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology. Many authors [6–10] studied several aspects of scalar field in the evolution of the universe and FRW models. Using the concept of Higgs field with potential $V(\phi)$ has a flat region and the field evolves slowly but the universe expands in an exponential way due to vacuum field energy [11]. It is assumed that the scalar field will take sufficient time cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. Recently, Bhattacharjee and Baruah [12], Bali and Jain [13], Rahaman et al. [14] and Reddy et al. [15] have studied the role of self-interacting scalar fields in inflationary cosmology in four-dimensional space-time.

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The study of higher dimensional space-time is important because of the underlying idea that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. This fact has attracted many researchers [16, 17] to the field of higher dimensions. Solutions of field equations in higher dimensional space-time are believed to be of physical relevance, possibly, at the early times before the universe has undergone compactification transitions. Further, Marciano [18] has suggested that experimental observation of fundamental constants with varying time could produce the evidence of extra dimensions.

In this paper, we have investigated a five dimensional inflationary cosmological model in the presence of massless scalar field with a flat potential in general relativity. To get a determinate solution, we have considered a flat region in which the potential is constant.

2 Metric and Field Equations

We consider the five dimensional Kaluza-Klein line element in the form

$$ds^2 = -dt^2 + R^2(t)\left[dx^2 + dy^2 + dz^2\right] + A^2(t)d\psi^2 \tag{1}$$

where the fifth coordinate is taken to be space-like.

In this case of gravity minimally coupled to a scalar field $V(\phi)$ the Lagrangian is [11]

$$L = \int \left[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\right]\sqrt{-g}d^4x \tag{2}$$

which on variations of L with respect to the dynamical fields leads to Einstein field equations

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{3}$$

with

$$T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right]g_{ij} \tag{4}$$

The field equation is

$$\phi^{i}_{;i} = -\frac{dV}{d\phi} \tag{5}$$

where comma and semicolon indicate ordinary and covariant differentiation respectively. Other symbols have their usual meaning and units are taken so that $8\pi G = c = 1$.

Now the Einstein field equations (3) for the metric (1) are given by

$$3\frac{R_4^2}{R^2} + 3\frac{R_4A_4}{RA} - \frac{\phi_4^2}{2\phi^2} + V(\phi) = 0 \tag{6}$$

$$2\frac{R_{44}}{R} + \frac{A_{44}}{A} + 2\frac{R_4A_4}{RA} + \frac{R_4^2}{R^2} - \frac{\phi_4^2}{2\phi^2} + V(\phi) = 0 \tag{7}$$

$$3\frac{R_{44}}{R} + 3\frac{R_4^2}{R^2} + \frac{\phi_4^2}{2\phi^2} + V(\phi) = 0 \tag{8}$$

and (5) for the scalar field takes the form

$$\phi_{44} + \phi_4\left(3\frac{R_4}{R} + \frac{A_4}{A}\right) - \frac{dV}{d\phi} = 0 \tag{9}$$

where the subscript 4 denotes differentiation with respect to t.

3 Solutions of the Field Equations and the Model

Since we are interested in inflationary solutions of the field equations, the flat region is considered where the potential is constant [11], i.e.,

$$V(\phi) = \text{constant} = V_0(\text{say}) \tag{10}$$

Now (9) can be, easily, integrated to give

$$AR^3\phi_4 = \phi_0 \tag{11}$$

where ϕ_0 is an integration constant.

From the field equations (6–8) with the help of (10), we get

$$\frac{R_{44}}{R} + 2\frac{R_4^2}{R^2} - 2\frac{R_4A_4}{RA} - \frac{A_{44}}{A} = 0 \tag{12}$$

Here we also assume the relation between the metric coefficients, i.e,

$$A = \mu R^n \tag{13}$$

because of the fact that the field equations are highly non-linear. Using the relation (13) the field equations (11) and (12) admit the exact solution

$$\begin{aligned} R &= \big[(n+3)(at+b)\big]^{1/n+3} \\ A &= \mu\big[(n+3)(at+b)\big]^{n/n+3} \\ \phi &= \frac{\phi_0}{a\mu}\left(\frac{1}{n+3}\right)\log\big[(n+3)(at+b)\big] \end{aligned} \tag{14}$$

where a and b are constants of integration. After a suitable choice of coordinates and integration constants the five dimensional inflationary cosmological model corresponding to the solution can be written as

$$ds^2 = -dT^2 + \big[(n+3)T\big]^{2/n+3}\big[dX^2 + dY^2 + dZ^2\big] + \mu^2\big[(n+3)T\big]^{2n/n+3}d\psi^2 \tag{15}$$

4 Some Physical Properties of the Model

The model (15) represents a five dimensional inflationary cosmological model in general relativity in the presence of massless scalar field with flat potential. The model has no initial singularity, i.e., at $T = 0$.

The physical and kinematical parameters for the model (15) are proper volume V^3, expansion scalar θ, shear scalar σ^2 and the deceleration parameter q and they have the following expressions for the five dimensional model given by (15):

$$V^3 = \sqrt{-g} = \mu(n+3)T \tag{16}$$

$$\theta = \frac{1}{3}u^i_{;i} = \frac{4}{(n+3)T} \tag{17}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{8}{3}\frac{1}{[(n+3)T]^2} \tag{18}$$

$$q = -3\theta^{-2}\left[\theta_{;i}u^i + \frac{1}{3\theta^2}\right] = -\frac{7}{4} \tag{19}$$

The spatial volume increases with time T and also when T tends to infinity, it approaches infinity. This shows that the model (15) is an inflationary universe with a massless scalar field in the flat region where the potential $V(\phi)$ is constant. Also the model inflates because of the fact that the deceleration parameter q is negative. The scalar expansion, the shear scalar vanish for large values of Tand they diverge for $T = 0$. Also, for $T = 0$ the scalar field ϕ diverges. Since

$$\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) = \frac{1}{6} \neq 0 \tag{20}$$

the model (15) does not approach isotropy for large values of T.

5 Conclusions

In this paper, we have obtained a five dimensional inflationary universe in the presence of massless scalar field with flat potential in general relativity. It is observed that the model is non-singular and does not approach isotropy for large T. Self-interacting scalar fields and higher dimensional cosmological models play a vital role in the study of early stages of evolution of the universe and we hope the model obtained will be useful for a better understanding of inflationary cosmology in higher dimensions.

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